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The Influence of the Resonant Frequency on the Presence Of Chimera State

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ABSTRACT
The Chimera State could be a result of the interaction between the resonant frequency and the synchronization process in a network of identical oscillators. The target of this paper is to do the numerically investigation of the chimera occurrence in a model with fifteen metronomes on each swing and two coupled swings Therefore, changing the value of metronomes oscillation frequency one can observe the level of synchronization between the two populations of metronomes through the Kuramoto complex order parameter. This analysis was conducted considering three different values of the connecting spring's stiffness among the swings. Thus, a relation between the presence of chimera state and the system resonant frequency was observed.
Keywords: Chimera States, Dynamic Systems, Resonance frequency, Synchronization

I. INTRODUCTION

Synchronization is a natural phenomenon that are present in many contexts, defining the rhythm of vital process of the organisms and in the flashing of fireflies [1] or in the atomic scale through the rhythmic vibrations of atoms making a laser beam. Could find too this phenomenon in simple mechanic dispositive like the metronomes [2-4], or eider in the dynamic of a bridge [5].

Christian Huygens was the first scientist that discuss about the synchronization phenomenon at 1665. After build the first pendulum clock, Huygens continue train improve it for develop a clock to maritime use. Therefore, he used a system with two coupled pendulums in the same base. He observed that the clocks oscillated in the same frequency in-phase or anti-phase.

In November of 2002, the Japanese physic Yoshiki Kuramoto, while he investigate a ring of coupled non-identical oscillators. He discovered something unexpected: for some initial conditions, oscillators that are equally coupled with its neighbors and has identical natural frequencies could differ each other in behavior [6]. That is, some oscillators can synchronize while others remain desynchronized. This is not transient, as a result of a asymmetric state of the initial conditions, but a permanent state that combine aspects from the synchronized state and the desynchronized. After, this state was called chimera state.

In Greek mythology, the chimera was a hybrid monster with lion's head, goat's body and serpent's tail [7]. However, in the non-linear dynamic chimera refer a mathematical surprising, a mix state of synchronized and desynchronized oscillators in a network of identically coupled oscillators. About one decade, the chimera state was observed just in numerical simulations [8-14]. Many of that chimeras depends of a carefully choose of the Initial conditions to stimulate sensible perturbations. Then were believed that stronger conditions as observed in experiments could make difficult to obtain the chimera state.

Just in 2012, this question was answered definitive when two experiments getting success to obtain chimeras with photochemistry oscillators [15,16]. The third experiment, made by Martens and his colleagues in 2013 [2], used coupled metronomes by a swing and two populations of metronomes coupled by a spring between the swings. The vibration of a swing make a strong coupled between the metronomes of the same swing and a weaker coupled between the swings. This was the first experiment that show the chimera state in a mechanical system.

II. PHYSICAL MODEL

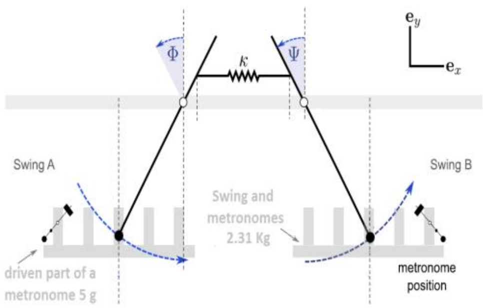


FIG 1. Physical model, adapted from Martens [2]

Martens [2] used fifteen metronomes by a swing to investigate the effect of varied of spring of strength coupling among the swings with the presence of the chimera states. Here is usage the mathematical model of Martens [2] with fifteen metronomes by swing. The system of equations of the model is:

$$\partial_\tau^2\Phi = \frac{\kappa}{\omega^2}(\Psi - \Phi) - \frac{\Omega^2}{\omega^2}\Phi - \mu_s\partial_\tau\Phi - \sum_{i=1}^{N}\partial_\tau^2 \sin\phi_i \qquad (1)$$

$$\partial_\tau^2\Psi = \frac{\kappa}{\omega^2}(\Phi - \Psi) - \frac{\Omega^2}{\omega^2}\Psi - \mu_s\partial_\tau\Psi - \sum_{i=1}^{N}\partial_\tau^2 \sin\psi_i \qquad (2)$$

$$\partial_\tau^2\phi_i = -\sin\phi_i - \mu_m\partial_\tau\phi_i\left(\left(\frac{\phi_i}{\theta_0}\right)^2 - 1\right) - \beta\cos\phi_i\,\partial_\tau^2\Phi \qquad (3)$$

$$\partial_\tau^2\psi_i = -\sin\psi_i - \mu_m\partial_\tau\psi_i\left(\left(\frac{\psi_i}{\theta_0}\right)^2 - 1\right) - \beta\cos\psi_i\,\partial_\tau^2\Psi \qquad (4)$$

Where the derivatives is with respective to time $\tau = \omega t$. The chimera states emerge from the competition among the synchronized and desynchronized state [2]. With the reduction of the spring of strength parameter κ is observed a transition between the synchronized state from in-phase to anti-phase. Martens [2] looking that close the amplitude-response curve there is a division among the in-phase and anti-phase behavior and which is in this region where the chimera states occur. Fact which create the hypothesis that chimera states has a relation with the resonance phenomenon [6].

This affirmation is based on the model of the coupled swings without the metronomes movement. What for the in-phase eigenmode has eigenfrequency Ω . Moreover, for the anti-phase (AP) the eingenmode has eigenfrequency:

$$\omega_{AP\ mode} = \sqrt{\Omega^2 + 2\kappa} \qquad (5)$$

The chimera state just was getting by the anti-phase eingenmode. For to measure how much desynchronized a coupled dynamic system is used the Kuramoto complex order parameter. Which if has value about 1 correspond a synchronized state, however for a value about 0 an incoherent state (desynchronized state).

$$Z_p(t) = K(t)e^{i\Theta(t)} = \frac{1}{N}\sum_{k=1}^{N} e^{i\left(\theta_k^p(t) - \bar{\theta}_{syn}(t)\right)} \qquad (6)$$

Where p denotes the population 1 or 2 that each oscillators are. N is the number of oscillators in each population and k is the number that show each oscillator of the population p. In addition $\bar{\theta}_{syn}$ represents the average phase of the synchronized population.

The simulation parameters of equations (1-4) are the same used by Pantaleone [3] and Martens [2], i. e., Ω = 6.67 rad/s-1, μ_s = 0.00016, μ_m = 0.011, β = 0.0005, θ = 0.33, κ and ω change to each simulation. And the initial conditions are the same adopts by Martens [2]. That are:

$$\phi_i(0) = \dot{\phi}_i(0) = 2\theta_0\left(r_i - \frac{1}{2}\right) \qquad (7)$$

Where i=1, 2, 3, ... N; r_i is a random number among [0,1]. To make sure of results of the simulation can create a chimera state; the both populations ought to have the same energy. That is:

$$\psi_i(0) = \frac{1}{N}\sum_{i}^{N}\phi_i(0) \qquad (9)$$

$$\dot{\psi}_i(0) = \frac{1}{N}\sum_{i}^{N}\dot{\phi}_i(0) \qquad (10)$$

In all the simulations the initial conditions relative to the swing movement was equal zero:

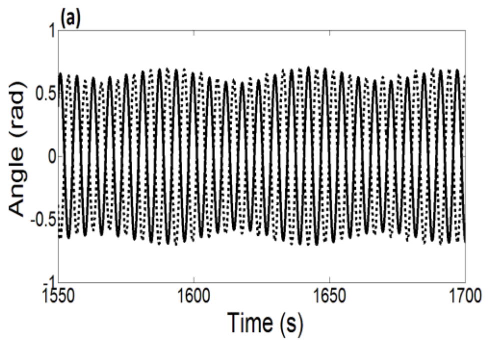
$$\Phi(0) = \Psi(0) = \dot{\Phi}(0) = \dot{\Psi}(0) = 0 \qquad (11)$$

All the results in this paper was obtained through the numerical integration of the equations (1-4) by the Runge-Kutta of 4th order with fixed time step dt = 0.013s and 2000 oscillations cycles. The permanent part is after 1500 oscillations.

III. RESULTS AND ANALYZE

Its possible verifies the presence of resonance through the position time evolution of the oscillators. If there is the resonance phenomenon, the signal of position time evolution demonstrate an increase in amplitude of oscillation or near the resonance the beating phenomenon is identified through a signal modulation.

Watching the figures 1(a-c) verify that the metronomes position time evolution show a modulation pattern, which is stronger in the chimera state. In figure 1(a) demonstrate the position time variation to in-phase state region, where show the synchronization between the both populations.



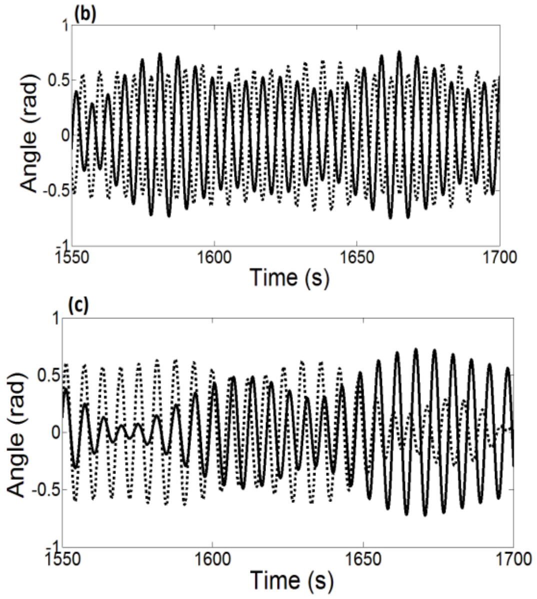
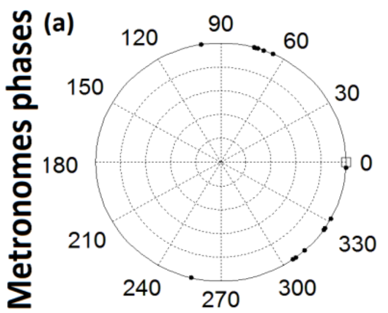


FIG 2. The dashed curve represent the movement of synchronized metronomes and the continuum curve a metronome of the incoherent population. And $\omega = 7$ rad/s is the natural frequency of the metronomes in all the cases. (a) Position time evolution to a spring strength among the swings of $\kappa = 0,6$ N/Kg.m. (b) Position time evolution to a spring strength among the swings of $\kappa = 2$ N/Kg.m. (c) Position time evolution to a spring strength among the swings of $\kappa = 2.5$ N/Kg.m.

In the figure 1(b) that indicate the metronomes position time evolution for the spring strength coupling of $\kappa = 2$ N/Kg.m observe a pattern of modulation. This value of the spring strength coupling are in the region of chimera state, but near the transition to the synchronized anti-phase state. This figure exhibit the modulation for the synchronized and incoherent population too.

Figure 1(c) show a strong modulation of synchronized population signal without miss the synchrony of the coherent population. The same pattern of modulation is observed to the desynchronized population. It is, the figure 1(c) demonstrate a result to a chimera state.



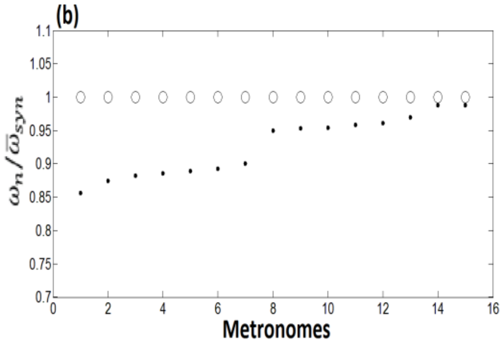


FIG 3. (a) Metronomes phases of the simulation showed in Fig. 1(c). The distribution of the phases of incoherent population is characteristic of the chimera states. The points denotes the phases of the incoherent population and the square is the phase of the synchronous population. (b) Relative frequencies between the two populations: coherent (circles) and incoherent (points). This variation between the frequencies of the two populations is a marked characteristic of chimera state.

Therefore, It's possible say that modulation is connected to chimera state. Because, the modulation became stronger when is looking near to the chimera state. And miss this characteristic of modulation when the system shows a synchronized state: in-phase or anti-phase.

The modulation is a marked characteristic of beating phenomenon, i. e., a forcing frequency near the natural frequency of the oscillation system. This is more one evidence that the chimera state is connected to the resonance phenomenon.

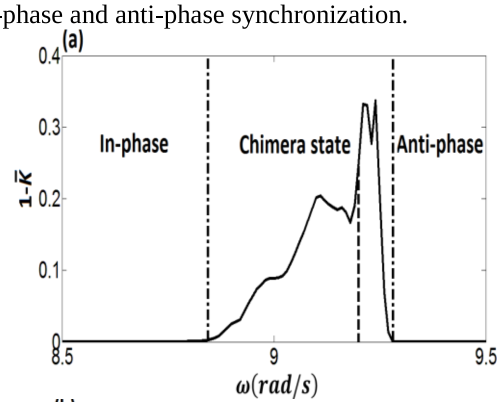
Investigating the possible relation between the chimera state and the resonance frequency, the amplitude-response curves was used to comprehend the appearing of the chimera state in relation of some values of natural frequency of the metronomes. But the amplitude-response curve as usually is, getting the points of bigger amplitude on time variation, no show correlation among the resonance and the chimera state. Because in the time variation of the metronomes don't have an increase on the amplitude, or neither a significant change of amplitudes from the synchronized case to the chimera state.

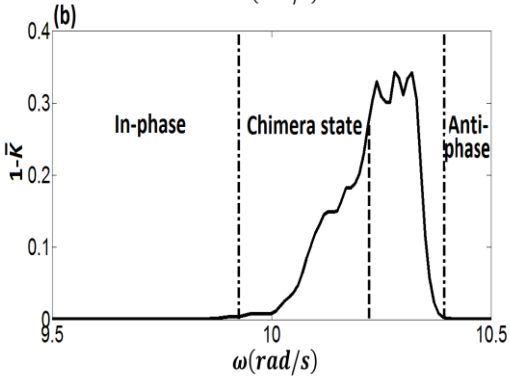
Therefore, was chosen a distinct parameter to describe this possible relation with the resonance. The chose parameter was the Kuramoto complex order parameter. However, it has a distribution on the complex plane and its value varying with the time. Then used its value as $z(t) = K(t)e^{\Theta(t)i}$, where Θ just define the angle in the complex plane and K the distance from the origin. Thus in the amplitude-response curve showed, the amplitude value is obtained by $1 - \bar{K}$; an average value to filter the time variation and subtracted this from one for the curve became near the characteristic amplitude-response curve.

Can verify in figure 3(a-c) that the peaks with bigger amplitude are dislocated to right of the resonance value of the system without the metronomes movement. Nevertheless, the peaks are near the resonance value and it is into the region of the chimera state. Exalting that down a limit obtain in-phase synchronization and for values up other limit obtain an anti-phase synchronization. And for value among these two limits find the chimera state.

That is, in the amplitude-response showed the peak that represent the presence of chimera state is very close of the resonance frequency of the system without the metronomes movement and in anti-phase eigenmode. Because that frequency is dominant in the system, as the force that move the metronomes through the swing motion. Hence, confirm a relation of dependency between the resonance frequency and the chimera state.

Watching the increase of the spring between the swings figures 4(a-c) is possible affirm that the chimera state region follow the value of the resonant frequency, showed by the dashed line on figures 4(a-c). This value change by the equation (5) and the chimera state follow it. Therefore, if is considered the plane of ω-κ and the axis $1-\bar{K}$ as a third dimension then the regions of higher desynchronization is fixed around the line of the resonant frequency as a wave dividing the regions of in-phase and anti-phase synchronization.





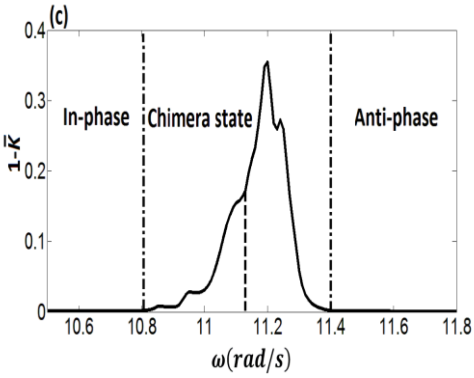


FIG 4. (a) Amplitude-response for $\kappa = 20$ N/Kg.m, the resonance frequency of the system without the metronomes movement is $\omega_{AP\ mode} = 9,20$ rad/s. (b) Amplitude-response for a spring strength among the swings of $\kappa = 30$ N/Kg.m, and resonance frequency of $\omega_{AP\ mode} = 10,22$ rad/s. (c) Amplitude-response for $\kappa = 40$ N/Kg.m, and resonance frequency of $\omega_{AP\ mode} = 11,16$ rad/s. The dashed line represents the resonance frequency to the system without the metronomes movement in all the cases.

The presence of the movement of metronomes in the analyze makes the system without an analytic solution for the system of equations of model and because of that, without a known natural frequency. The metronomes are the forced elements and due the inertia of the swing with the metronomes (2.31 Kg) in comparison with the inertia just the driven part of the metronomes (5 g by metronome; and 75 g of all metronomes by swing). Know which the oscillation frequency of all the system (with the driven part of the metronomes in the analyze) just get a small displacement in relation of the system without the metronomes movement.

This displacement is demonstrate in figures 3 (a-c). Therefore, the peaks in figures 3(a-c) are find in a probable place for the value of system resonance as all, including the metronomes movement. The beating phenomenon shows this closeness too.

IV. CONCLUSION

Thus, with these facts infer that the presence of the chimera state depends of the resonance phenomenon. That is, the chimera state is a manifestation of the proximity of the resonance frequency in globally coupled oscillators with identical natural frequencies in the synchronization process.

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